Armand Noubissie

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the late afternoon

Title:
UNIVERSAL SKOLEM SET AND ULTIMATE POSITIVITY PROBLEM.

Abstract:
Let S be a subset of N. We say that S is a simple Skolem set if there is an effective procedure that, given a simple linear recurrence sequence denoted LRSS, decides whether the sequence has a zero in S . When a such procedure holds for all LRS, we say S is a Skolem set. In the first part of this talk, we give an explicit example of Skolem set with positive density and we will show in the second part that the Ultimate positivity problem is decidable under one condition based on the Theory of Diophantine Approximation. This is a joint work in progress with F. Luca, J. Ouaknine and J. Worrell.

Bruno Salvy

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the late afternoon

Title:
Positivity certificates for P-recursive sequences

Abstract:

TBA

Clara Lacroe

Arriving: Thursday, 27th April in the afternoon (will stay first night in Bridgetown)
Leaving: Thursday, 4th May in the late afternoon

Title:
An AAK theory approach to the approximate minimization problem.

Abstract:
In this talk I will analyze the approximate minimization problem for weighted finite automata (WFAs). I will start by introducing the Adamyan-Arov-Krein approximation theory, a collection of results on the approximation of Hankel operators. Then, I will show how this theory can be leveraged to provide a closed-form solution and an algorithm to compute the optimal approximation of a given size to a WFA defined over a one-letter alphabet. Finally, I will highlight the main obstacles towards a generalization of this approach to the multi-letter setting.

David Purser

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the late afternoon

Title:
Linear dynamical systems under floating-point rounding

Abstract:
We consider linear dynamical systems under floating-point rounding. In these systems, a matrix is repeatedly applied to a vector, but the numbers are rounded into floating-point representation after each step (i.e., stored as a fixed-precision mantissa and an exponent). The approach more faithfully models realistic implementations of linear loops, compared to the exact arbitrary-precision setting often employed in the study of linear dynamical systems.

Our results are twofold: We show that for non-negative matrices there is a special structure to the sequence of vectors generated by the system: the mantissas are periodic and the exponents grow linearly. We leverage this to show decidability of ω-regular temporal model checking against semialgebraic predicates. This contrasts with the unrounded setting, where even the non-negative case encompasses the long-standing open Skolem and Positivity problems.

On the other hand, when negative numbers are allowed in the matrix, we show that the reachability problem is undecidable by encoding a two-counter machine. Again, this is in contrast with the unrounded setting where point-to-point reachability is known to be decidable in polynomial time.
Joint work with Engel Lefaucheux, Joël Ouaknine, and Mohammadamin Sharifi

Emre Sertöz

Arriving: Thursday, 27th April in the afternoon
Leaving: Thursday, 4th May in the late afternoon

Title:
Separating period integrals of quartic surfaces

Abstract:
Periods form a natural number system that extends the algebraic numbers by adding values of integrals coming from geometry and physics. Because there are countably many periods, one would expect it to be possible to compute effectively in this number system. This would require an effective height function and the ability to separate periods of bounded height, neither of which are even remotely possible in general. I will, however, introduce a separation constant to numerically verify identities coming from integrals related to quartic surfaces in 3-space, i.e., related to K3 surfaces. This is joint work with Pierre Lairez.

Florian Luca

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the afternoon

Title:
Universal Skolem Sets

Abstract:
Any non-degenerate linearly recurrent sequence has only finitely many zeros. This is the celebrated Skolem-Mahler-Lech theorem. We know bounds on how many of them can be but not how large they can be. Bounds on how large the zeros can be are known only if the sequence is very special.
Rather than imposing restrictions on the sequence we imposed restrictions on the integers on which we sample linearly recurrent sequences for zeros. This lead to the notion of a Universal Skolem Set, a set on which we can compute the zeros of any non-degenerate linearly recurrent sequence. In my talk, I will tell the story of how we came up with this concept, what the first example looked like, where we are now, and where we hope to be going in the near future.
Coauthors: J. Ouaknine and J. Worrell

James Ben Worrell

Arriving: Friday, 28th April in the late afternoon
Leaving: Friday, 5th May in the late evening

Title:
Transcendence of Sturmian numbers over an algebraic base

Abstract:
We consider numbers of the form \sum_{n=0}^\infty u_n/b^n for (u_n) a Sturmian sequence over a binary alphabet and b an algebraic number of absolute value greater than one. We show that every such number is transcendental. More generally, for a given base b and given irrational number θ, we prove Q-linear independence of the set comprising 1 together with all numbers of the above form whose associated digit sequences have slope θ.

We give an application of our main result to the theory of dynamical systems. We show that for a Cantor set C arising as the set of limit points of a contracted rotation f on the unit interval, where f is assumed to have an algebraic slope, all elements of C except its endpoints 0 and 1 are transcendental.

This is joint work with Florian Luca and Joel Ouaknine.

Joël Ouaknine

Arriving: Wednesday, 26th April in the afternoon
Leaving: Thursday, 4th May in the evening

Title:
Skolem meets Schanuel

Abstract:
I will discuss how simple p-adic techniques enable one to solve the Skolem Problem for simple linear recurrence sequences (LRS). More precisely, there is an algorithm which takes as input a simple (no repeated characteristic root), non-degenerate integer LRS, and computes its (necessarily finite) set of zeros. The algorithm moreover outputs a certificate of correctness, certifying the absence of zeros (other than the ones computed). The termination of this algorithm relies on two classical number-theoretic conjectures, the exponential local-global principle, and the p-adic Schanuel conjecture. The tool is implemented and can be freely experimented with online at https://skolem.mpi-sws.org/ .

Joris Nieuwveld

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the late afternoon

Title:
The Skolem Problem and rational zeros

Abstract:
The Skolem Problem, which asks to decide whether a given linear recurrence sequence (LRS) has a zero, has been historically approached with Baker's Theorem on linear forms in logarithms. In recent times, new techniques have been developed based on looking at the sequence modulo well-chosen numbers and isolating subsequences using p-adics. These ideas led to a breakthrough: Only needing some conjectures for termination, one can decide the Skolem Problem for simple LRS. For non-simple LRS, this approach cannot work. In this talk, I will showcase a refinement of these new techniques using rational zeros. Such zeros of an LRS occur not at integer indices n such that u_n = 0 but at r for some rational number r. I will discuss how to define such zeros, some general problems involving them, and how to use them to solve specific instances of the Skolem Problem.

Lutz Klingenberg

Arriving: Thursday, 27th April in the afternoon
Leaving: Saturday, 13th May in the late afternoon

Title:
Generation Functions in Probabilistic Programs

Abstract:
We study discrete probabilistic programs with potentially unbounded looping behaviors over an infinite state space. We present, to the best of our knowledge, the first decidability result for the prob- lem of determining whether such a program generates exactly a specified distribution over its outputs (provided the program terminates almost-surely). The class of distributions that can be specified in our formalism consists of standard distributions (geometric, uniform, etc.) and finite convolutions thereof. Our method relies on representing these (possibly infinite-support) distributions as probability generating functions which admit effective arithmetic operations.

Marcel Moosbrugger

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the afternoon

Title:
Probabilistic Loop Analysis with Recurrences

Abstract:
Probabilistic programming languages enrich classical programming languages with native primitives to draw samples from random distributions, such as Bernoulli, Uniform, and Normal distributions. Loops, like in the automated analysis of classical programs, are the main obstacle to probabilistic program analysis. Loops can be seen as discrete-time dynamical systems, while probabilistic loops encode stochastic processes. Probabilistic loops can model processes ranging from biological systems to privacy protocols. Our recent paper proposes a method that models moments of loop variables, such as expected value and variance, as linear recurrences with constant coefficients. Such recurrences always admit computable closed forms (such as exponential polynomials). With the closed forms of moments of program variables, our method can automatically find a basis for all polynomial invariants among a finite set of variable moments. Other use cases include computing bounds on tail probabilities or estimating the distributions of loop variables. Our method is sound and complete for a class of probabilistic loops that can be characterized syntactically.

Martina Maggio

Arriving: Friday, 28th April in the late afternoon
Leaving: Saturday, 6th May in the evening

Title:
Analysis of linear time-invariant systems with and without linear controllers

Abstract:
Controllers are tyically used to regulate the behaviour of dynamical systems. Very often, a physical system is described with ordinary differential equations, and then linearised and discretised around an equilibrium point, in order to design a controller that is supposed to keep the system at that equilibrium point. However, when the controller is implemented, there might be computational problems (signals that are not received) or security attacks (malicious actions that prevent the controller from executing correctly). Even when this does not happen, the designer may want to use different controllers to deal with complex scenarios, that expose different trade offs. In the talk, we will explore the tools that are available to analyse these systems, and highlight their shortcomings and open research questions.

Mihir Vahanwala

Arriving: Friday, 28th April in the afternoon
Leaving: Friday, 5th May in the late afternoon

Title:
On Robustness for Linear Recurrence Sequences

Abstract:
The Skolem, Positivity, and Ultimate Positivity problems for Linear Recurrence Sequences (LRS) are number-theoretic problems whose decidability has been open for decades. They are known to be at least as hard as Diophantine approximation, an open number-theoretic problem. With scientific applications ranging from theoretical biology to software verification and formal languages, LRS are a fundamental mathematical primitive. Given the inherent imprecision and need for safety margins in the real world, we consider the problems for LRS with the following notion of robustness: does the sequence satisfy the required property despite small perturbations in the given initialisation? Although some interpretations of this notion yield problems that are still Diophantine hard, others can be shown to be decidable, in PSPACE even! In this talk, we discuss how the decision procedure strings together remarkable and profound results from computational algebra, number theory, and logic. Indeed, we shall summarise the motivation and the modern template to tame Linear Recurrence Sequences.

Ruiwen Dong

Arriving: Friday, 28th April in the late afternoon
Leaving: Friday, 5th May in the evening

Title:
Decidability problems in infinite semigroups

Abstract:
This talk is about several algorithmic problems for sub-semigroups of infinite groups. Working in an ambient group G (usually a matrix group), we consider the following decision problems.

(i) Semigroup Membership: Given a finite subset A of G and an element g of G, does g belong to the semigroup generated by A?
(ii) Semigroup Intersection: Given two finite subsets A and B of G, do the two semigroups generated respectively by A and B have empty intersection?
(iii) Group Problem: Given a finite subset A of G, is the semigroup generated by A a group?

All three problems are known to be undecidable for general groups following classic results dating from the 1940s. Current active research mainly focuses on proving (un)decidability or complexity results for specific classes of groups G.

In the first part of the talk, we will give a quick survey of current known results on these problems in specific classes of groups, notably abelian groups, nilpotent groups, solvable groups and general matrix groups. The main tools range from automata and language theory to Lie algebra and algebraic geometry.

In the second part of the talk, we will give a sketch of a new result showing decidability of the Group Problem in metabelian groups G. These are groups whose commutator is abelian (a.k.a. 2-step solvable groups). The main idea combines group theory, graphs, convex geometry, algebraic geometry, and culminates in a local-global principle concerning linear equations over the polynomial semiring $N[X_1, ..., X_n]$.

Rupak Majumdar

Arriving: Friday, 28th April in the afternoon
Leaving: Thursday, 4th May in the late afternoon

Title:
General decidability results in concurrent system verification under context bounds

Abstract:
I will give a survey on general decidability results on context-bounded analysis of concurrent systems. These are discrete-time, discrete-state, dynamical systems arising out of algorithmic program verification, and the techniques to obtain decidability results are language-theoretic. I will state an open problem in the analysis of these systems related to the analysis of probabilistic systems: I would like to enlist your help in showing decidability, undecidability, or a connection to a different hard problem.

Toghrul Karimov

Arriving: Friday, 28th April in the afternoon
Leaving: Saturday, 6th May in the late afternoon

Title:
The Model-Checking Problem for Linear Dynamical Systems

Abstract:
A discrete-time LDS is given by a $d \times d$ update matrix M and a starting configuration x. The orbit of the LDS (M, x) is the infinite sequence $(x, Mx, M^2x, ...)$. We consider the following model-checking problem: given an LDS (M, x), a semi-algebraic partition $P = \{T_1, \ldots, T_m\}$ of $\mathbb{R}^d$, and an automaton A over P, determine if the word $\alpha \in P^\omega$ induced by the orbit of M is accepted by A. We discuss the main techniques for proving decidability of the model-checking problem for various classes of LDS.

Valérie Berthé

Arriving: Friday, 28th April in the afternoon
Leaving: Tuesday, 9th May in the afternoon

Title:
Strong convergence for infinite products of matrices.

Abstract:
We discuss convergence properties for infinite products of matrices with nonnegative entries.
One motivation comes from the study of continued fractions. A multidimensional continued fraction is expected to yield simultaneous better and better rational approximations with the same denominator for given d-tuples of real numbers. The main advantage of most classical unimodular continued fractions is that they can be expressed as dynamical systems whose ergodic study has already been well understood. The main disadvantage relies in the quality of approximation in higher dimensions. Indeed, it is governed by Lyapounov exponents which describe the asymptotic behaviour of the singular values of large products of random matrices, under the ergodic hypothesis. In a joint work with W. Steiner and J. Thuswaldner, we have very recently noticed experimentally that the second Lyapunov exponent is not even negative in higher dimensions for the most classical algorithms such as the Jacobi-Perron, Brun or Selmer algorithms, which prevents strong convergence of these algorithms.